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MODELING OF HEAT OUTFLOWS TO A PROBE IN THERMOPHYSICAL TESTING

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The nondestructive method developed by the author with the use of a linear pulsed heat source makes it possible to determine the thermophysical characteristics of solid materials on the basis of the portions of thermograms that correspond to the regularization of the thermal regime in the region of the heater and temperature detectors. Consideration has been given to the influence of the outflows of heat to a probe on the error in determining the thermophysical characteristics of materials in the case where two semiinfinite bodies are in ideal contact. A mathematical expression upon the fulfillment of which one may disregard the heat loss to the material of the probe substrate has been obtained. In the work, consideration has also been given to the problem of heat loss to the probe material in the case where it occurs only in the region of the heater. It has been shown that the temperatures of the first and second bodies in the contact area will become closer with time.

The determination of thermophysical characteristics is based on the physical model presented in Fig. 1. Thermal action on the body under study with a uniform initial temperature distribution is carried out using a linear pulsed heater. In the experiment, the temperature is recorded at a prescribed distance from the heater [1].

The theoretical foundations of the method of nondestructive testing of the thermophysical characteristics of materials, which uses the model of a nonstationary process of heat transfer from a linear pulsed heat source, have been presented in [1]. The method allows for different states of operation of a measuring system. The analytical solution of the mathematical model of the process of heat transfer in the body under study from the action of a linear pulsed source for the second ("operating") portion of the thermogram has the form [2]

$$T(r,\tau)=\frac{q_0}{2\pi\lambda}\left(\ln\frac{4a\tau}{r^2}-\gamma\right). \tag{1}$$

Let q_{10} and q_{20} be parts of the power going to heat the material under study and the material of the probe substrate respectively. The condition when the outflows of heat to the probe material are negligible may be written in the form

$$\frac{q_{20}}{q_{10}}<<1\ . \tag{2}$$

To evaluate q_{20}/q_{10} we consider the following problem.

Two semiinfinite bodies are in ideal thermal contact (Fig. 2). A linear heat source of constant power in the form of a band of width $2h$ acts in the contact plane. The power released per unit area of the heater is equal to $\overline{q}_0$ (or, in terms of the power per unit length of the heater, to $\overline{q}_0 = q_0/2h$). Then the temperature field in this system at any instant of time will be determined by solution of the following mathematical problem:

$$\frac{1}{a_1}\frac{\partial T_1(x,y,\tau)}{\partial\tau}=\frac{\partial^2T_1(x,y,\tau)}{\partial x^2}+\frac{\partial^2T_1(x,y,\tau)}{\partial y^2}\,,\quad \tau>0\,,\ \ -\infty<x<\infty\,,\ \ y>0\,; \tag{3}$$

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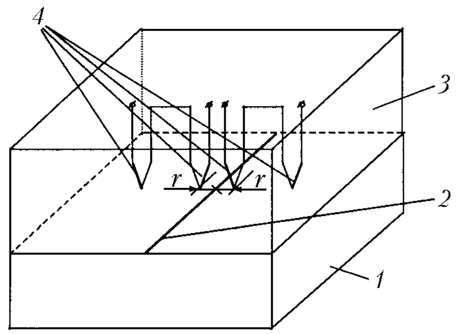

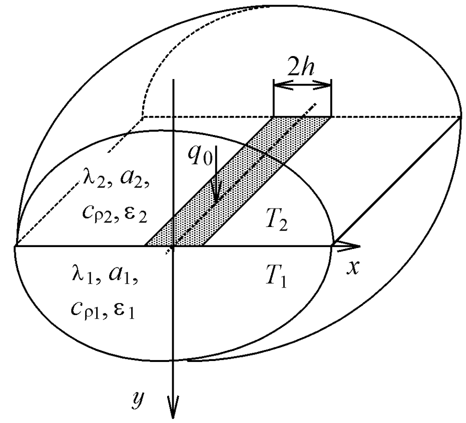

Fig. 1. Diagram of the thermal system for the method with a linear pulsed heater: 1) sample under study; 2) heater; 3) measuring probe; 4) thermocouples.

Fig. 2. Diagram for the linear heater in the form of a band, acting in the plane of contact of two semiinfinite bodies.

$$\frac{1}{a_2}\frac{\partial T_2(x,y,\tau)}{\partial \tau}=\frac{\partial^2 T_2(x,y,\tau)}{\partial x^2}+\frac{\partial^2 T_2(x,y,\tau)}{\partial y^2}\,,\quad \tau>0\,,\quad -\infty<x<\infty\,,\quad y<0\,; \tag{4}$$

$$T_1(x,y,0)=T_2(x,-y,0)=0\,; \tag{5}$$

$$T_1(x,\infty,\tau)=T_1(\infty,y,\tau)=T_2(x,-\infty,\tau)=T_2(\infty,-y,\tau)=0\,; \tag{6}$$

$$\frac{\partial T_1(\infty,y,\tau)}{\partial x}=\frac{\partial T_1(x,\infty,\tau)}{\partial y}=\frac{\partial T_2(\infty,-y,\tau)}{\partial x}=\frac{\partial T_2(x,-\infty,\tau)}{\partial y}=0\,; \tag{7}$$

$$T_1(x,+0,\tau)=T_2(x,-0,\tau)\,; \tag{8}$$

$$-\lambda_1\frac{\partial T_1(x,+0,\tau)}{\partial y}-\lambda_2\frac{\partial T_2(x,-0,\tau)}{\partial y}=f(x)\,; \tag{9}$$

$$f(x)=\begin{cases}\dfrac{q_0}{2h}\,, & |x|<h\,,\\ 0\,, & |x|>h\,;\end{cases} \tag{10}$$

$$\frac{\partial T_1(0,y,\tau)}{\partial x}=\frac{\partial T_2(0,-y,\tau)}{\partial x}=0\,. \tag{11}$$

We integrate the equations of system (3)–(11) with respect to $x \in (-\infty, \infty)$. For Eq. (3) we have

$$\int_{-\infty}^{+\infty}\frac{1}{a_1}\frac{\partial T_1(x,y,\tau)}{\partial \tau}\,dx=\int_{-\infty}^{+\infty}\frac{\partial^2 T_1(x,y,\tau)}{\partial x^2}\,dx+\int_{-\infty}^{+\infty}\frac{\partial^2 T_1(x,y,\tau)}{\partial y^2}\,dx\,,$$

$$\frac{1}{a_1}\frac{\partial\int\limits_{-\infty}^{+\infty}T_1(x,y,\tau)\,dx}{\partial\tau}=\frac{\partial^2\int\limits_{-\infty}^{+\infty}T_1(x,y,\tau)\,dx}{\partial y^2}+2\int\limits_0^{+\infty}\frac{\partial^2 T_1(x,y,\tau)}{\partial x^2}dx\,,$$

$$\int\limits_0^{+\infty}\frac{\partial^2 T_1(x,y,\tau)}{\partial x^2}dx=\left.\frac{\partial T_1(x,y,\tau)}{\partial x}\right|_{x=0}^{\infty}=\begin{pmatrix}\text{see (7)}\\ (11)\end{pmatrix}=0\,,$$

$$\frac{1}{a_1}\frac{\partial\int\limits_{-\infty}^{+\infty}T_1(x,y,\tau)\,dx}{\partial\tau}=\frac{\partial^2\int\limits_{-\infty}^{+\infty}T_1(x,y,\tau)\,dx}{\partial y^2}\,. \qquad (12)$$

Analogously, for Eqs. (4)–(10) we obtain

$$\frac{1}{a_2}\frac{\partial\int\limits_{-\infty}^{+\infty}T_2(x,y,\tau)\,dx}{\partial\tau}=\frac{\partial^2\int\limits_{-\infty}^{+\infty}T_2(x,y,\tau)\,dx}{\partial y^2}\,, \qquad (13)$$

$$\int\limits_{-\infty}^{+\infty}T_1(x,y,0)\,dx=\int\limits_{-\infty}^{+\infty}T_2(x,y,0)\,dx=0\,, \qquad (14)$$

$$\int\limits_{-\infty}^{+\infty}T_1(x,\infty,\tau)\,dx=\int\limits_{-\infty}^{+\infty}T_2(x,-\infty,\tau)\,dx=0\,, \qquad (15)$$

$$\frac{\partial\int\limits_{-\infty}^{+\infty}T_1(x,\infty,\tau)\,dx}{\partial y}=\frac{\partial\int\limits_{-\infty}^{+\infty}T_2(x,-\infty,\tau)\,dx}{\partial y}=0\,, \qquad (16)$$

$$\int\limits_{-\infty}^{+\infty}T_1(x,0,\tau)\,dx=\int\limits_{-\infty}^{+\infty}T_2(x,0,\tau)\,dx\,; \qquad (17)$$

$$-\int\limits_{-\infty}^{+\infty}\left(\lambda_1\frac{\partial T_1(x,0,\tau)}{\partial y}+\lambda_2\frac{\partial T_2(x,0,\tau)}{\partial y}\right)dx=\int\limits_{-\infty}^{+\infty}f(x)\,dx\,,\quad \int\limits_{-\infty}^{+\infty}f(x)\,dx=\int\limits_{-h}^{+h}\frac{q_0}{2h}dx=\left.\frac{q_0}{2h}x\right|_{x=-h}^{h}=q_0\,,$$

$$-\lambda_1\frac{\partial\int\limits_{-\infty}^{+\infty}T_1(x,0,\tau)\,dx}{\partial y}-\lambda_2\frac{\partial\int\limits_{-\infty}^{+\infty}T_2(x,0,\tau)\,dx}{\partial y}=q_0\,. \qquad (18)$$

We introduce the following integral characteristics:

$$S_1(y, \tau) = \int_{-\infty}^{+\infty} T_1(x, y, \tau)\, dx , \tag{19}$$

$$S_2(y, \tau) = \int_{-\infty}^{+\infty} T_2(x, y, \tau)\, dx . \tag{20}$$

With account for dependences (12)–(20), we may rewrite system (3)–(11) in terms of the new functions $S_1(y, \tau)$ and $S_2(y, \tau)$ in the form

$$\frac{1}{a_1} \frac{\partial S_1(y, \tau)}{\partial \tau} = \frac{\partial^2 S_1(y, \tau)}{\partial y^2} , \quad \tau > 0 , \quad y > 0 ; \tag{21}$$

$$\frac{1}{a_2} \frac{\partial S_2(y, \tau)}{\partial \tau} = \frac{\partial^2 S_2(y, \tau)}{\partial y^2} , \quad \tau > 0 , \quad y < 0 ; \tag{22}$$

$$S_1(y, 0) = S_2(—y, 0) = 0 ; \tag{23}$$

$$S_1(\infty, \tau) = S_2(-\infty, \tau) = 0 ; \tag{24}$$

$$\frac{\partial S_1(\infty, \tau)}{\partial y} = \frac{\partial S_2(-\infty, \tau)}{\partial y} = 0 ; \tag{25}$$

$$S_1(+0, \tau) = S_2(-0, \tau) ; \tag{26}$$

$$-\lambda_1 \frac{\partial S_1(+0, \tau)}{\partial y} - \lambda_2 \frac{\partial S_2(-0, \tau)}{\partial y} = q_0 . \tag{27}$$

We consider the physical meaning of the expressions $-\lambda_1 \frac{\partial S_1(+0, \tau)}{\partial y}$ and $-\lambda_2 \frac{\partial S_2(-0, \tau)}{\partial y}$.

Taking into account that $-\lambda_1 \frac{\partial T_1(x, +0, \tau)}{\partial y}$ is the heat flux arriving at the first body, $-\lambda_2 \frac{\partial T_2(x, -0, \tau)}{\partial y}$ is the heat flux arriving at the second body, and $-\lambda_1 \frac{\partial S_1(+0, \tau)}{\partial y} = -\int_{-\infty}^{\infty} \lambda_1 \frac{\partial T_1(x, +0, \tau)}{\partial y} dx$ and $-\lambda_2 \frac{\partial S_2(-0, \tau)}{\partial y} = -\int_{-\infty}^{\infty} \lambda_2 \frac{\partial T_2(x, -0, \tau)}{\partial y} dx$ are the quantities of power going to heat the first and second bodies respectively, we may write

$$-\lambda_1 \frac{\partial S_1(+0, \tau)}{\partial y} = q_{10} , \quad -\lambda_2 \frac{\partial S_2(-0, \tau)}{\partial y} = q_{20} . \tag{28}$$

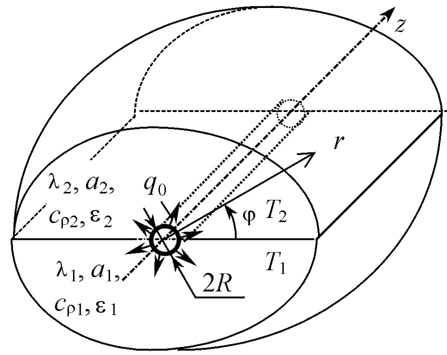

Fig. 3. Diagram for the heater in the form of a cylinder, acting in the plane of contact of two semiinfinite bodies.

Thus, solving system (21)–(27) and subsequently determining $-\lambda_1 \frac{\partial S_1(+0, \tau)}{\partial y}$ and $-\lambda_2 \frac{\partial S_2(-0, \tau)}{\partial y}$, we obtain the condition upon the fulfillment of which we may disregard the heat loss to the probe-substrate material. Also, it is noteworthy that the dimension of the heater is not involved in this system and the right-hand side of expression (27) involves the quantity q_0. It may be inferred that system (21)–(27) will describe the processes for an ideal linear heat source ($2h \to 0$), too.

Problem (21)–(27) is equivalent to that on heating of two semiinfinite bodies at the site of whose contact a plane heat source of constant power acts [3]. The solution of such a problem may be written in the form

$$S_1(y, \tau) = \frac{2q_0\sqrt{\tau}}{\varepsilon_1 + \varepsilon_2} \operatorname{ierfc}\left[\frac{y}{2\sqrt{a_1\tau}}\right], \quad S_2(y, \tau) = \frac{2q_0\sqrt{\tau}}{\varepsilon_1 + \varepsilon_2} \operatorname{ierfc}\left[\frac{y}{2\sqrt{a_2\tau}}\right]. \tag{29}$$

Consequently, we have

$$q_{10} = -\lambda_1 \frac{\partial S_1(+0, \tau)}{\partial y} = \frac{q_0 \varepsilon_1}{\varepsilon_1 + \varepsilon_2}, \quad q_{20} = -\lambda_2 \frac{\partial S_2(-0, \tau)}{\partial y} = \frac{q_0 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}. \tag{30}$$

With account for (2), we obtain the expression upon the fulfillment of which we may disregard the heat loss to the probe-substrate material:

$$\frac{q_{20}}{q_{10}} = \frac{\varepsilon_2}{\varepsilon_1} << 1 . \tag{31}$$

It has been assumed that the first and second bodies are in ideal thermal contact. However, in actual practice, we will always have thermal resistances between the probe and the sample under study in the contact plane. They will be much lower in the region of the heater than those in the region of contact of the probe-substrate material and the material under study, i.e., condition (31) will be overstated. In this connection, it is of interest to obtain expression (2) on condition that heat is lost to the probe-substrate material only in the heater region.

We find the stationary field of heat fluxes in the probe–heater–sample system on condition that heat is lost to the probe-substrate material only in the region of the heater.

Two semiinfinite bodies are in contact. Their contacting surfaces are heat-insulated. A linear heat source of constant power in the form of a cylinder of radius R acts in the contact plane (Fig. 3).

The power released per unit area of the heater is equal to $\bar{q}_0$ (or, in terms of the power per unit length of the heater, to $\bar{q}_0 = \frac{q_0}{2\pi R}$). Then the temperature field at any instant of time (on condition that the temperature gradient along the z axis is absent) in this system will be determined by solution of the following mathematical problem:

$$\frac{\partial T_1(r,\varphi,\tau)}{\partial\tau}=a_1\left(\frac{\partial^2 T_1(r,\varphi,\tau)}{\partial r^2}+\frac{1}{r}\frac{\partial T_1(r,\varphi,\tau)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T_1(r,\varphi,\tau)}{\partial\varphi^2}\right),$$

$$r>R\,,\ -\frac{\pi}{2}\le\varphi<0\,,\ \tau>0\,; \tag{32}$$

$$\frac{\partial T_2(r,\varphi,\tau)}{\partial\tau}=a_2\left(\frac{\partial^2 T_2(r,\varphi,\tau)}{\partial r^2}+\frac{1}{r}\frac{\partial T_2(r,\varphi,\tau)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T_2(r,\varphi,\tau)}{\partial\varphi^2}\right),$$

$$r>R\,,\ 0<\varphi\le\frac{\pi}{2}\,,\ \tau>0\,; \tag{33}$$

$$T_1(r,\varphi,0)\Big|_{\substack{r\ge R\\ -\frac{\pi}{2}\le\varphi\le 0}}=0\,,\quad T_2(r,\varphi,0)\Big|_{\substack{r\ge R\\ 0\le\varphi\le\frac{\pi}{2}}}=0\,; \tag{34}$$

$$T_1(R,\varphi,\tau)\Big|_{\substack{\tau>0\\ -\frac{\pi}{2}\le\varphi\le 0}}=T_2(R,\varphi,\tau)\Big|_{\substack{\tau>0\\ 0\le\varphi\le\frac{\pi}{2}}}\,; \tag{35}$$

$$T_1(\infty,\varphi,\tau)\Big|_{\substack{\tau>0\\ -\frac{\pi}{2}\le\varphi<0}}=T_2(\infty,\varphi,\tau)\Big|_{\substack{\tau>0\\ 0<\varphi\le\frac{\pi}{2}}}=0\,; \tag{36}$$

$$\frac{\partial T_1(r,\varphi,\tau)}{\partial\varphi}\Big|_{\substack{\varphi=-0\\ r>R\\ \tau>0}}=\frac{\partial T_2(r,\varphi,\tau)}{\partial\varphi}\Big|_{\substack{\varphi=+0\\ r>R\\ \tau>0}}=0\,,\quad \frac{\partial T_1(r,\varphi,\tau)}{\partial\varphi}\Big|_{\substack{\varphi=-\frac{\pi}{2}\\ r>R\\ \tau>0}}=\frac{\partial T_2(r,\varphi,\tau)}{\partial\varphi}\Big|_{\substack{\varphi=\frac{\pi}{2}\\ r>R\\ \tau>0}}=0\,; \tag{37}$$

$$-\lambda_1\frac{\partial T_1(R,\varphi,\tau)}{\partial r}\Big|_{\substack{\tau>0\\ -\frac{\pi}{2}\le\varphi<-0}}=\bar{q}_{10}\,; \tag{38}$$

$$-\lambda_2\frac{\partial T_2(R,\varphi,\tau)}{\partial r}\Big|_{\substack{\tau>0\\ +0<\varphi\le\frac{\pi}{2}}}=\bar{q}_{20}\,; \tag{39}$$

$$\pi R\bar{q}_{10}+\pi R\bar{q}_{20}=q_0\,. \tag{40}$$

Under the assumption that the temperature gradient in each of the semiinfinite bodies in question is independent of the coordinate φ and with account for condition (37), we obtain a problem equivalent to that given above. Equations (32) and (33) with the corresponding initial and boundary conditions will be written as follows:

$$\frac{\partial T_1(r,\tau)}{\partial\tau}=a_1\left(\frac{\partial^2 T_1(r,\tau)}{\partial r^2}+\frac{1}{r}\frac{\partial T_1(r,\tau)}{\partial r}\right),\quad \tau>0\,,\quad r>R\,,\quad -\frac{\pi}{2}\le\varphi<0\,; \tag{41}$$

$$\frac{\partial T_2(r,\tau)}{\partial \tau} = a_2\left(\frac{\partial^2 T_2(r,\tau)}{\partial r^2} + \frac{1}{r}\frac{\partial T_2(r,\tau)}{\partial r}\right), \quad \tau > 0\,, \quad r > R\,, \quad 0 < \varphi \le \frac{\pi}{2}\,; \tag{42}$$

$$T_1(r,0)\Big|_{\substack{r \ge R \\ -\frac{\pi}{2} \le \varphi \le 0}} = 0\,, \quad |T_2(r,0)\Big|_{\substack{r \ge R \\ 0 \le \varphi \le \frac{\pi}{2}}} = 0\,; \tag{43}$$

$$T_1(R,\tau)\Big|_{\substack{\tau > 0 \\ -\frac{\pi}{2} \le \varphi \le 0}} = T_2(R,\tau)\Big|_{\substack{\tau > 0 \\ 0 \le \varphi \le \frac{\pi}{2}}}\,; \tag{44}$$

$$T_1(\infty,\tau)\Big|_{\substack{\tau > 0 \\ -\frac{\pi}{2} \le \varphi < 0}} = T_2(\infty,\tau)\Big|_{\substack{\tau > 0 \\ 0 < \varphi \le \frac{\pi}{2}}} = 0\,; \tag{45}$$

$$-\lambda_1 \frac{\partial T_1(R,\tau)}{\partial r}\Big|_{\substack{\tau > 0 \\ -\frac{\pi}{2} \le \varphi < -0}} = \bar{q}_{10}\,; \tag{46}$$

$$-\lambda_2 \frac{\partial T_2(R,\tau)}{\partial r}\Big|_{\substack{\tau > 0 \\ +0 < \varphi \le \frac{\pi}{2}}} = \bar{q}_{20}\,; \tag{47}$$

$$\pi R \bar{q}_{10} + \pi R \bar{q}_{20} = q_0\,. \tag{48}$$

Applying the Laplace transformation to (41) and (42), with account for (43) we obtain

$$\frac{d^2 T_{1L}(r,p)}{dr^2} + \frac{1}{r}\frac{dT_{1L}(r,p)}{dr} - \frac{p}{a_1} T_{1L}(r,p) = 0\,, \tag{49}$$

$$\frac{d^2 T_{2L}(r,p)}{dr^2} + \frac{1}{r}\frac{dT_{2L}(r,p)}{dr} - \frac{p}{a_2} T_{2L}(r,p) = 0\,. \tag{50}$$

The general solutions of the differential equations (49) and (50) (Bessel equations) have the form [4]

$$T_{1L}(r,p) = A_1 I_0\left[\sqrt{\frac{p}{a_1}}\, r\right] + B_1 K_0\left[\sqrt{\frac{p}{a_1}}\, r\right], \tag{51}$$

$$T_{2L}(r,p) = A_2 I_0\left[\sqrt{\frac{p}{a_2}}\, r\right] + B_2 K_0\left[\sqrt{\frac{p}{a_2}}\, r\right]. \tag{52}$$

Expression (36) yields that $A_1 = A_2 = 0$. Then formulas (51) and (52) are transformed as

$$T_{1L}(r,p) = B_1 K_0\left[\sqrt{\frac{p}{a_1}}\, r\right], \tag{53}$$

$$T_{2L}(r,p) = B_2 K_0\left[\sqrt{\frac{p}{a_2}}\, r\right]. \tag{54}$$

The coefficients B_1 and B_2 will be determined from conditions (46)–(48):

$$\pi R\lambda_1 B_1 \sqrt{\frac{p}{a_1}}\, K_1\left[\sqrt{\frac{p}{a_1}}\, r\right] + \pi R\lambda_2 B_2 \sqrt{\frac{p}{a_2}}\, K_1\left[\sqrt{\frac{p}{a_2}}\, r\right] = \frac{q_0}{p}, \quad B_1 K_0\left[\sqrt{\frac{p}{a_1}}\, r\right] = B_2 K_0\left[\sqrt{\frac{p}{a_2}}\, r\right].$$

Solving this system for B_1 and B_2, we obtain

$$B_1 = \frac{q_0 K_0\left[\sqrt{\frac{p}{a_2}}\, R\right]}{\pi R p^{3/2}\left(\varepsilon_1 K_1\left[\sqrt{\frac{p}{a_1}}\, R\right] K_0\left[\sqrt{\frac{p}{a_2}}\, R\right] + \varepsilon_2 K_1\left[\sqrt{\frac{p}{a_2}}\, R\right] K_0\left[\sqrt{\frac{p}{a_1}}\, R\right]\right)}, \tag{55}$$

$$B_2 = \frac{q_0 K_0\left[\sqrt{\frac{p}{a_1}}\, R\right]}{\pi R p^{3/2}\left(\varepsilon_1 K_1\left[\sqrt{\frac{p}{a_1}}\, R\right] K_0\left[\sqrt{\frac{p}{a_2}}\, R\right] + \varepsilon_2 K_1\left[\sqrt{\frac{p}{a_2}}\, R\right] K_0\left[\sqrt{\frac{p}{a_1}}\, R\right]\right)}. \tag{56}$$

Substituting expressions (55) and (56) into formulas (53) and (54), we obtain the solution of problem (41)–(48) in the domain of Laplace transformations:

$$T_1(r,p) = \frac{q_0 K_0\left[\sqrt{\frac{p}{a_1}}\, r\right] K_0\left[\sqrt{\frac{p}{a_2}}\, R\right]}{\pi R p^{3/2}\left(\varepsilon_1 K_1\left[\sqrt{\frac{p}{a_1}}\, R\right] K_0\left[\sqrt{\frac{p}{a_2}}\, R\right] + \varepsilon_2 K_1\left[\sqrt{\frac{p}{a_2}}\, R\right] K_0\left[\sqrt{\frac{p}{a_1}}\, R\right]\right)}, \tag{57}$$

$$T_2(r,p) = \frac{q_0 K_0\left[\sqrt{\frac{p}{a_2}}\, r\right] K_0\left[\sqrt{\frac{p}{a_1}}\, R\right]}{\pi R p^{3/2}\left(\varepsilon_1 K_1\left[\sqrt{\frac{p}{a_1}}\, R\right] K_0\left[\sqrt{\frac{p}{a_2}}\, R\right] + \varepsilon_2 K_1\left[\sqrt{\frac{p}{a_2}}\, R\right] K_0\left[\sqrt{\frac{p}{a_1}}\, R\right]\right)} \tag{58}$$

and expressions for determination of the field of heat fluxes:

$$\bar{q}_{10}(r,p) = \frac{q_0 \varepsilon_1 K_1\left[\sqrt{\frac{p}{a_1}}\, r\right] K_0\left[\sqrt{\frac{p}{a_2}}\, R\right]}{\pi R p^{3/2}\left(\varepsilon_1 K_1\left[\sqrt{\frac{p}{a_1}}\, R\right] K_0\left[\sqrt{\frac{p}{a_2}}\, R\right] + \varepsilon_2 K_1\left[\sqrt{\frac{p}{a_2}}\, R\right] K_0\left[\sqrt{\frac{p}{a_1}}\, R\right]\right)}, \tag{59}$$

$$\bar{q}_{20}(r,p)=\frac{q_0\varepsilon_2K_1\left[\sqrt{\frac{p}{a_2}}\,r\right]K_0\left[\sqrt{\frac{p}{a_1}}\,R\right]}{\pi Rp^{3/2}\left(\varepsilon_1K_1\left[\sqrt{\frac{p}{a_1}}\,R\right]K_0\left[\sqrt{\frac{p}{a_2}}\,R\right]+\varepsilon_2K_1\left[\sqrt{\frac{p}{a_2}}\,R\right]K_0\left[\sqrt{\frac{p}{a_1}}\,R\right]\right)}\,. \quad (60)$$

Using the limiting theorem [3], we determine the stationary heat-flux distribution in the system in question:

$$\bar{q}_{10}(r)=\lim_{\tau\to\infty}\bar{q}_{10}(r,\tau)=\lim_{p\to 0}\,[p\bar{q}_{10}(r,p)]=$$

$$=\lim_{p\to 0}\left(\frac{q_0\varepsilon_1K_1\left[\sqrt{\frac{p}{a_1}}\,r\right]K_0\left[\sqrt{\frac{p}{a_2}}\,R\right]}{\pi R\left(\varepsilon_1K_1\left[\sqrt{\frac{p}{a_1}}\,R\right]K_0\left[\sqrt{\frac{p}{a_2}}\,R\right]+\varepsilon_2K_1\left[\sqrt{\frac{p}{a_2}}\,R\right]K_0\left[\sqrt{\frac{p}{a_1}}\,R\right]\right)}\right),$$

$$\bar{q}_{10}(r)=\frac{q_0\lambda_1}{\pi r\,(\lambda_1+\lambda_2)}\,. \quad (61)$$

Analogously, for expression (60) we may write

$$\bar{q}_{20}(r)=\frac{q_0\lambda_2}{\pi r\,(\lambda_1+\lambda_2)}\,. \quad (62)$$

The powers arriving to heat bodies 1 and 2 will be determined from the formulas

$$q_{10}=\pi R\bar{q}_{10}(R)=\frac{q_0\lambda_1}{\lambda_1+\lambda_2}\,, \quad (63)$$

$$q_{20}=\pi R\bar{q}_{20}(R)=\frac{q_0\lambda_2}{\lambda_1+\lambda_2}\,, \quad (64)$$

and condition (2) will take the form

$$\frac{q_{20}}{q_{10}}=\frac{\lambda_2}{\lambda_1}<<1\,. \quad (65)$$

Equations (61) and (62) make it possible to find the temperature distribution in the sample and the probe relative to the heater temperature (there is no steady-state temperature field in this system). In the case were $r = R$, we have

$$T_1(R,\tau)=T_2(R,\tau)=T_{\rm h}(\tau)\,,$$

where $T_{\rm h}(\tau)$ is the heater temperature. Taking into account that

$$\bar{q}_{10}(r)=-\lambda_1\frac{\partial T_1(r,\tau)}{\partial r} \quad (66)$$

and it is determined from (61), we may write, previously integrating (66) from R to r, for the first and second bodies

$$T_1 (r, \tau) = T_h (\tau) - \frac{q_0}{\pi (\lambda_1 + \lambda_2)} \ln \left[\frac{r}{R} \right], \tag{67}$$

$$T_2 (r, \tau) = T_h (\tau) - \frac{q_0}{\pi (\lambda_1 + \lambda_2)} \ln \left[\frac{r}{R} \right]. \tag{68}$$

As is seen from expressions (67) and (68), the temperatures of the first and second bodies in the region of contact will become closer with time.

NOTATION

A_1, A_2, B_1, and B_2, coefficients; a_1 and a_2, thermal conductivities of the material under study and the probe-substrate material respectively, m^2/sec; $c_{\rho 1}$ and $c_{\rho 2}$, heat capacity per unit volume of the material under study and the probe-substrate material, J/(K·m^3); $2h$, bandwidth of the heater, m; $I_0(x)$ and $K_0(x)$, modified Bessel functions of the first and second kind of zero order; p, complex variable; q_0, power released per unit length of the heater, W/m; q_{10} and q_{20}, powers going to heat the material under study and the probe-substrate material, W/m; $\bar{q}_0$, power released per unit area of the heater, W/m^2; $\bar{q}_{10}$ and $\bar{q}_{20}$, power going to heat the material under study and the probe-substrate material, W/m^2; R, radius, m; $S_1(y, \tau)$ and $S_2(y, \tau)$, integral temperatures of the planes which are in parallel to the plane of contact of two bodies and pass through the points with a coordinate y, K; T, temperature, K; x, y, z, r, coordinates, m; φ, angle, rad; $\gamma = 0.5772$, Euler number; τ, time, sec; ε_1 and ε_2, thermal activity of the material under study and the probe-substrate material, W·sec$^{0.5}$·m^{-2}·K^{-1}; λ, thermal conductivity, W/(m·K); λ_1 and λ_2, thermal conductivities of the material under study and the probe-substrate material. Subscripts: h, heater; 1, material under study; 2, material of the probe substrate; L, functions transformed according to Laplace.

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